

Stability Derivatives from Aircraft Geometry

Longitudinal and Lateral-Directional Building Blocks

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Longitudinal Building Blocks (from wing/tail geometry)

$$C_{L_\alpha} = a_w \left(1 + \frac{S_{HT}}{S} \frac{a_t}{a_w} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right)$$

$$C_{L_q} = 2a_t \bar{V}_H, \quad C_{L_{\dot{\alpha}}} = 2a_t \bar{V}_H \frac{\partial \epsilon}{\partial \alpha}$$

$$C_{m_\alpha} = C_{L_\alpha} (h - h_{nwb}) - \eta_t a_t \bar{V}_H \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

$$C_{m_q} = -2a_t \bar{V}_H \frac{l_t}{\bar{c}}, \quad C_{m_{\dot{\alpha}}} = -2a_t \bar{V}_H \frac{l_t}{\bar{c}} \frac{\partial \epsilon}{\partial \alpha}$$

$$C_{D_\alpha} = 2K_{C_L} C_{L_e} C_{L_\alpha}$$

where a_w, a_t are the wing/tail lift-curve slopes, $\bar{V}_H = \frac{S_{HT} l_t}{S \bar{c}}$ is the horizontal tail volume ratio, h, h_{nwb} are the cg and wing-body neutral point (as fractions of $\bar{c}$), and K_{C_L} is the induced-drag factor ($C_D = C_{D_0} + K_{C_L} C_L^2$).

Total Lift

$$\begin{aligned}
 L &= L_{wb} + L_t \\
 C_L = \frac{L}{qS} &= \frac{C_{L_{wb}} qS}{qS} + \frac{C_{L_t} q_t S_t}{qS} \\
 C_L &= C_{L_{wb}} + \frac{S_t}{S} \eta C_{L_t}
 \end{aligned}$$

$$\begin{aligned}
 C_L &= C_{L_{wb}} + \frac{S_t}{S} \eta C_{L_t} \\
 &= a_{wb} \alpha_{wb} + \frac{S_t}{S} \eta a_t (\alpha_{wb} - i_t - \epsilon) \\
 &= a_{wb} \alpha_{wb} + \frac{S_t}{S} \eta a_t \left[\alpha_{wb} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) - i_t - \epsilon_0 \right] \\
 &= \underbrace{-a_t \frac{S_t}{S} \eta (i_t + \epsilon_0)}_{C_{L_0}} + \underbrace{a_{wb} \left[1 + \frac{a_t S_t}{a_{wb} S} \eta \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right]}_a \alpha_{wb}
 \end{aligned}$$

Total Pitching Moment

$$C_m = C_{m_0} + C_{m_\alpha} \alpha$$

$$C_{m_0} = C_{m_{ac_{wb}}} + a_t \bar{V}_H \eta (i_t + \epsilon_0) \left[1 - \frac{a_t}{a} \frac{S_t}{S} \eta \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right]$$

$$C_{m_\alpha} = a(h - h_{n_{wb}}) - \bar{V}_H \eta a_t \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

Neutral Point

Note that C_{m_α} can be made negative (positive stiffness) by a suitable choice of h (position of CG). The *neutral point* is defined as the CG position h_n such that $C_{m_\alpha} = 0$:

$$h_n = h_{n_{wb}} + \eta \frac{a_t}{a} \bar{V}_H \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

Lateral-Directional Building Blocks (from geometry)

$$C_{Y_\beta} = -\eta_{vt} \frac{S_{VT}}{S} a_{vt} \left(1 - \frac{\partial \sigma}{\partial \beta}\right)$$

$$C_{n_\beta} = \eta_{vt} \frac{S_{VT}}{S} \frac{l_F}{b} a_{vt} \left(1 - \frac{\partial \sigma}{\partial \beta}\right)$$

$$C_{l_\beta} = -\frac{z_F}{b} \frac{S_{VT}}{S} \eta_{vt} a_{vt} \left(1 - \frac{\partial \sigma}{\partial \beta}\right)$$

$$C_{n_r} = -\eta_{vt} a_{vt} V_v \left(2\hat{l}_F + \frac{\partial \hat{\sigma}}{\partial \hat{r}}\right)$$

$$C_{l_r} = \eta_{vt} a_{vt} \frac{S_{VT}}{S} \frac{z_F}{b} \left(2\hat{l}_F + \frac{\partial \hat{\sigma}}{\partial \hat{r}}\right)$$

$$C_{Y_p} = -\eta_{vt} a_{vt} \frac{S_{VT}}{S} \left(2\hat{z}_F + \frac{\partial \hat{\sigma}}{\partial \hat{r}}\right)$$

$$C_{n_p} = \eta_{vt} a_{vt} V_v \left(2\hat{z}_F + \frac{\partial \hat{\sigma}}{\partial \hat{r}}\right)$$

where $\hat{l}_F = l_F/b$, $\hat{z}_F = z_F/b$, $V_v = \eta_{vt} \frac{S_{VT}}{S} \frac{l_F}{b}$ is the vertical tail volume ratio, and a_{vt} is the vertical-tail lift-curve slope. All results are referred to $\mathcal{F}_B$ about the centre of mass, with l_F and z_F measured as body-axis distances, so no axis conversion follows. **No simple geometric formula exists** for C_{l_p} , C_{Y_r} , or any control derivative – these require a vortex-lattice/panel tool (e.g. AVL) or wind-tunnel/flight data.

Estimation of C_{n_β}

► Fuselage (Body)

$$\underbrace{(C_n)_{body}}_{(-)} = \underbrace{(C_{n_\beta})_{body}}_{(-)} \beta$$

► Vertical Tail (fin)

$$(C_{n_\beta})_F = \frac{S_F}{S} \frac{l_F}{b} \eta a_F (1 - \frac{\partial \sigma}{\partial \beta})$$

► Wing (Sweep)

$$(C_{n_\beta})_{w,\Lambda} = C_D \bar{y} / b \sin(2\Lambda)$$

In summary,

$$C_{n_\beta} = \underbrace{(C_{n_\beta})_{body}}_{(-)} + \underbrace{(C_{n_\beta})_F}_{(+)} + \underbrace{(C_{n_\beta})_{w,\Lambda}}_{(+)} > 0$$

2. Lateral-Directional Derivatives from Geometry

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figures/rudder

The rudder is installed at the trailing edge of the vertical tail (fin), where the additional side force (therefore, the corresponding yawing moment) can be generated by deflecting the rudder surface,

$$(C_y)_F = -a_F \alpha_F + a_r \delta_r$$

Question 1. What is the impact of rudder deflection on yaw stability?

2. Lateral-Directional Derivatives from Geometry

The rudder is installed at the trailing edge of the vertical tail (fin), where the additional side force (therefore, the corresponding yawing moment) can be generated by deflecting the rudder surface,

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figures/rudder

$$(C_y)_F = -a_F \alpha_F + a_r \delta_r$$

To consolidate into the system, using the reference $1/2\rho V^2 S b$, we will get the yawing moment control derivative:

$$C_{n_{\delta_r}} = \frac{\partial C_{n_F}}{\partial \delta_r} = -V_V \left(\frac{V_F}{V} \right)^2 \frac{\partial C_{y_F}}{\partial \delta_r} = -\eta a_r V_V, \quad \text{where } V_V = \frac{l_F S_F}{b S}$$

2. Lateral-Directional Derivatives from Geometry

When the aircraft is banked from its symmetric flight, as shown below, the motion of aircraft may be affected.

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figures/rollstiffness

$$C_{l_\phi} = \frac{\partial C_l}{\partial \phi} < 0$$

ϕ comes from sideslip and AoA:

$$\phi \approx \beta \cdot \alpha$$

leading to

$$C_{l_\phi} = C_{l_\beta} \cdot \alpha$$

- ▶ self study: aileron control

Aileron Control

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