

Basic Control Systems Review

Models, Feedback, and Response

Acknowledgement: These slides were developed with reference to course materials by Professor Hugh H. T. Liu.

How to Use This Material

- ▶ This reading reviews models, stability, transient response, frequency response, and basic compensators used in classical feedback flight control.
- ▶ It is a refresher from introductory controls; work through each checkpoint before proceeding to controller design.
- ▶ Connect each mathematical object to its physical meaning, units, operating condition, and modelling assumptions.

Review Map

Models and Representations

Feedback Fundamentals

Stability and Time Response

Frequency Response

Basic Controller Actions

Practical Workflow

A Control System Starts with Signals

Common signals

- ▶ command $r(t)$ and error $e(t)$
- ▶ control input $u(t)$ and output $y(t)$
- ▶ disturbance $d(t)$ and noise $n(t)$

Design questions

- ▶ Is the response stable, fast, and well damped?
- ▶ Is steady-state error acceptable?
- ▶ Are robustness and control effort acceptable?

LTI Input–Output Models

An n th-order, single-input single-output LTI model can be written as

$$a_0 y^{(n)} + \cdots + a_n y = b_0 u^{(m)} + \cdots + b_m u.$$

With zero initial conditions, the Laplace transform gives

$$G(s) \triangleq \frac{Y(s)}{U(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}.$$

- ▶ Poles are denominator roots; zeros are numerator roots.
- ▶ A transfer function describes one input–output map, not the internal state uniquely.
- ▶ Linearized aircraft models are local approximations about a trim condition.

State Space and Transfer Functions

$$\dot{\underline{x}} = A\underline{x} + B\underline{u},$$

$$\underline{y} = C\underline{x} + D\underline{u}.$$

$$G(s) = C(sI - A)^{-1}B + D.$$

Pole-zero cancellation can hide a mode in a particular input-output map without removing the internal state.

The eigenvalues of A describe natural modes. State space handles multiple inputs and outputs directly.

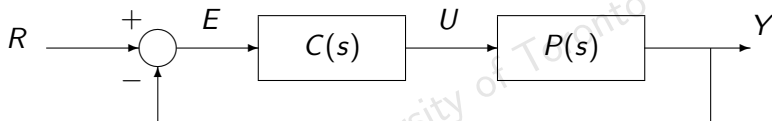
Checkpoint: declare states, inputs, outputs, frames, units, and the validity range of the model.

Standard Test Inputs

Input	Time domain	Laplace transform
Impulse	$\delta(t)$	1
Step	$1(t)$	$1/s$
Ramp	$t1(t)$	$1/s^2$
Sinusoid	$\sin \omega t$	$\omega/(s^2 + \omega^2)$

The impulse response characterizes an LTI system; the step response exposes tracking, speed, damping, and offset; a sinusoid measures frequency response at one frequency.

The Negative-Feedback Loop



$$L = CP, \quad T = \frac{Y}{R} = \frac{L}{1 + L}, \quad S = \frac{E}{R} = \frac{1}{1 + L}.$$

The identity $S + T = 1$ expresses a central tradeoff: feedback cannot make both sensitivity functions small at the same frequency.

What Feedback Does

With an output disturbance d and measurement noise n ,

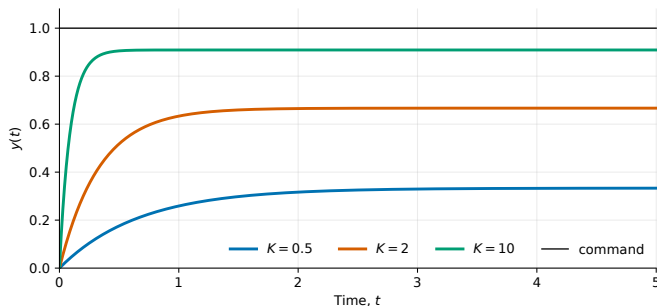
$$Y = TR + SD - TN.$$

- ▶ Large $|L|$ at low frequency usually improves tracking, rejects slow disturbances, and reduces sensitivity to plant uncertainty.
- ▶ Small $|L|$ at high frequency usually limits sensor-noise amplification and improves robustness to unmodelled fast dynamics.
- ▶ The crossover region sets the main bandwidth and robustness tradeoff.

For $P(s) = 1/(s + 1)$ and $C(s) = K$,

$$T(s) = \frac{K}{s + 1 + K}, \quad e_{ss} = \frac{1}{1 + K}.$$

Feedback Gain: Benefit and Cost



Increasing K reduces offset and speeds up this simple model. Real limits include saturation, noise, delay, uncertainty, and neglected dynamics.

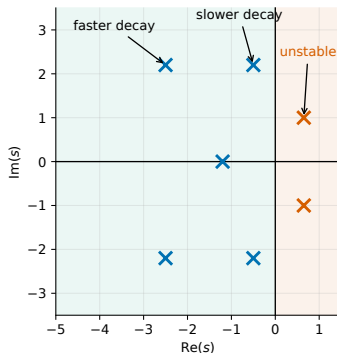
Closed-Loop Stability

Closed-loop poles are roots of $1 + L(s) = 0$. For a continuous-time finite-dimensional LTI system:

- ▶ all poles in the open left half-plane imply asymptotic stability;
- ▶ any pole in the open right half-plane implies instability;
- ▶ repeated imaginary-axis poles imply instability;
- ▶ simple imaginary-axis poles, with all others stable, imply marginal stability.

Pole calculation, Routh–Hurwitz, root locus, and Nyquist analysis offer complementary stability tests. Stable plant poles alone do not guarantee a stable feedback loop.

Pole Location Predicts Motion

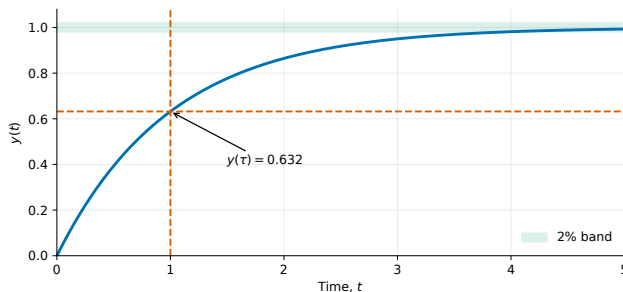


For $p = \sigma + j\omega$, the associated motion contains $e^{\sigma t}$ and oscillates at $|\omega|$. For a complex pair,

$$\omega_n = |p|, \quad \zeta = -\frac{\Re(p)}{|p|}, \quad \omega_d = \omega_n \sqrt{1 - \zeta^2}.$$

First-Order Response

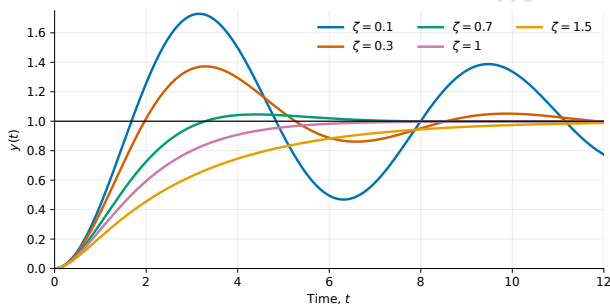
For $G(s) = 1/(\tau s + 1)$, the unit-step response is $y(t) = 1 - e^{-t/\tau}$.



At $t = \tau$, the response reaches 63.2%; the 2% settling time is approximately 4τ .

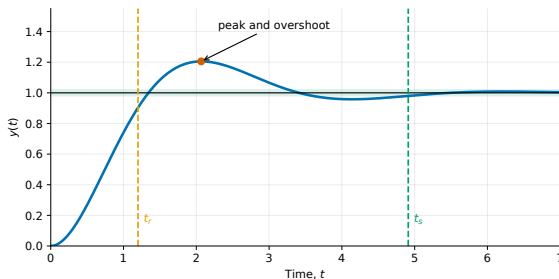
Canonical Second-Order Response

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$



Larger ω_n sets a faster time scale; increasing ζ generally reduces oscillation and overshoot.

Step-Response Specifications



For an underdamped canonical second-order system,

$$M_p = 100 \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) \%,$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}, \quad t_s (2\%) \approx \frac{4}{\zeta \omega_n}.$$

Rise-time definitions vary; state the threshold convention.

Dominant Poles and Model Reduction

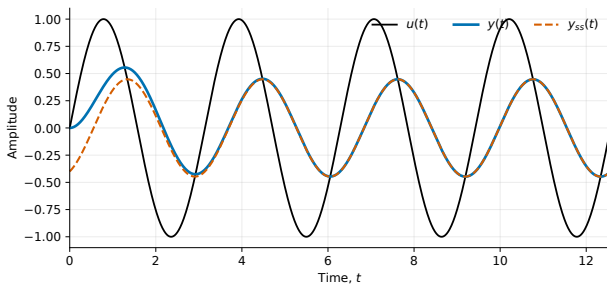
- ▶ Poles nearest the imaginary axis often dominate because their transients decay most slowly.
- ▶ Neglect a far-left pole only when its time scale is well separated and its residue is not large.
- ▶ A near cancellation can hide a mode in one transfer function without removing its internal dynamics.
- ▶ Nonminimum-phase zeros can cause inverse response and limit achievable speed.

Short-period, phugoid, roll, spiral, and Dutch-roll modes can be interpreted through pole location, damping, frequency, and modal participation.

Sinusoidal Steady-State Response

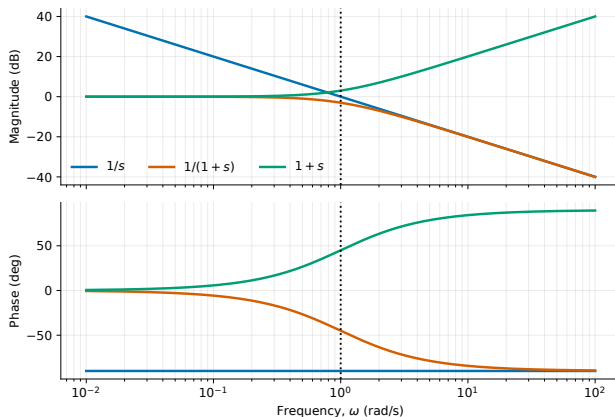
For stable $G(s)$ and $u(t) = U_0 \sin \omega t$,

$$y_{ss}(t) = |G(j\omega)| U_0 \sin(\omega t + \angle G(j\omega)).$$



The complete response also contains a transient determined by the poles and initial conditions.

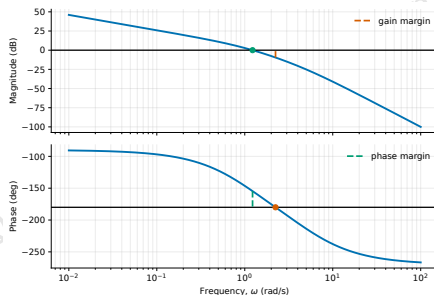
Reading a Bode Plot



An integrator contributes -20 dB/decade and -90° ; a real pole adds about -20 dB/decade and -90° after its break; a real zero adds the opposite. Decibel magnitudes and phases add when factors multiply.

Bandwidth, Resonance, and Margins

For a low-pass closed-loop response, $|T(j\omega_{BW})| = |T(0)|/\sqrt{2}$ commonly defines bandwidth. Higher bandwidth usually means faster tracking, but it must remain below uncertain dynamics and noise-dominated frequencies.



$$PM = 180^\circ + \angle L(j\omega_{gc}), \quad GM = 1/|L(j\omega_{pc})|.$$

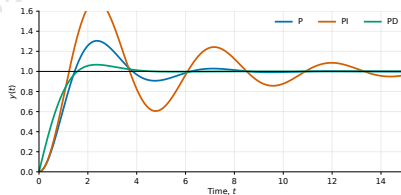
Margins are indicators; multiple crossovers, delay, unstable open-loop poles, and uncertainty require fuller analysis.

P, I, and D Actions

$$C(s) = K_p + \frac{K_i}{s} + K_d s.$$

Action	Main benefit	Main caution
P	immediate correction	offset may remain
I	removes constant offset	windup and phase lag
D	adds damping/phase lead	amplifies noise

Derivative action is filtered in practice, and integral action requires anti-windup when actuators saturate.



Lead, Lag, and Loop Shaping

Lead:

$C(s) = K(1 + s/z)/(1 + s/p)$,
 $z < p$. Adds positive phase and
can increase crossover and
phase margin.

Lag:

$C(s) = K(1 + s/z)/(1 + s/p)$,
 $p < z$. Raises low-frequency
gain relative to crossover, with
a phase penalty.

PID, lead-lag, notch, and low-pass filters are combinations of poles and zeros selected to shape $L(s)$ while respecting noise, robustness, and actuator constraints.

From Requirements to a Control Design

1. Define the operating condition, signals, units, limits, disturbances, noise, and quantitative requirements.
2. Obtain and validate a plant model over the intended amplitude and frequency range.
3. Check poles, zeros, modes, and uncertain or neglected dynamics.
4. Select an architecture and shape low-, crossover-, and high-frequency loop gain.
5. Verify stability, margins, time response, offset, control effort, and saturation recovery.
6. Test robustness across uncertainty and operating conditions before higher-fidelity simulation or flight test.

Review Checklist

Before classical flight-control design, be able to

- ▶ move among ODE, state-space, transfer-function, pole-zero, and frequency-response descriptions;
- ▶ form L , S , and T for a negative-feedback loop;
- ▶ infer stability and transient behaviour from poles;
- ▶ interpret step specifications and Bode plots;
- ▶ qualify gain margin, phase margin, and bandwidth;
- ▶ explain P, I, D, lead, and lag action and their limitations;
- ▶ assess robustness, noise sensitivity, saturation, and control effort together.